

Design of Advanced PID Controller for Enhanced Disturbance Rejection of Second-Order Processes with Time Delay

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*A design method for a proportional-integral-derivative (PID) cascaded with a lead-lag compensator is proposed for enhanced disturbance rejection of the second-order stable and unstable processes with time delay. A two-degree-of-freedom control scheme is used to cope with both regulatory and servo problems. An ideal feedback controller equivalent to internal model control (IMC) is obtained through the IMC design principle, and is further simplified to the PID cascaded with a first-order lead-lag compensator. The simulation is conducted for a broad class of stable and unstable processes and the results are compared with those of recently published PID type controllers to illustrate the superiority of the proposed controller. For a reasonable comparison, the controllers in the simulation study are tuned to have the same degree of robustness by measuring the maximum sensitivity, M_s . The robustness of the controller is also investigated by simultaneously inserting a perturbation uncertainty in all parameters in order to obtain the worst-case model mismatch. The proposed method illustrates greater robustness against process parameter uncertainty. © 2008 American Institute of Chemical Engineers *AIChE J.* 54: 1526–1536, 2008*

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Introduction

It is common practice to reduce the actual process to a low-order model because most tuning rules for the proportional-integral-derivative (PID) type industrial controller are based on such low-order models. It is also well known that the second-order plus time delay model represents the dynamics of the actual process better than the first-order plus time delay model for various processes in chemical process industries. Furthermore, in chemical process industries, the majority of control loops is of the PID type due to its relatively simple structure, which can be easily understood and implemented in practice.

To find a simple design method of the PID type controller with a significant performance improvement has become an important research issue for process engineers. Because of the simplicity and improved performance of the internal model control (IMC)-based tuning rule, the analytically derived IMC-PID tuning methods have attracted the attention of industrial users over the last decade. The IMC-PID tuning rule has only one user-defined tuning parameter, which is directly related to the closed-loop time constant. The IMC-PID tuning methods^{1–7} and the direct synthesis (DS) method^{8,9} are two examples of typical tuning methods based on achieving a desired closed-loop response. These methods obtain the PID controller parameters by computing the controller which gives the desired closed-loop response. Although this controller is often more complicated than a PID controller, its form can be reduced to that of either a PID only controller or a PID cascaded with a low-order lag

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filter by some clever approximations of the dead time in the process model.

The IMC-PID controller provides good set-point tracking but sluggish disturbance response, especially for processes with a small time-delay/time-constant ratio. However, for many process control applications, disturbance rejection is much more important than set-point tracking. Several researchers^{1–9} have reported that the suppressing load disturbance is poor when the process dynamics are significantly slower than the desired closed-loop dynamics. Therefore, a controller design emphasizing disturbance rejection rather than set-point tracking is an important design problem that has received renewed interest recently.

Furthermore, it is well known that the control system design for an open-loop, unstable process is even more difficult especially in the presence of a time delay. Therefore, some modified IMC methods^{10–14} of two-degree-of-freedom (2DOF) control were developed for controlling unstable processes with time delay. Several tuning methods^{15–17} have been proposed based on the Smith-Predictor (SP) to achieve a smooth nominal set-point response without overshoot for first-order unstable processes with time delay. Both the modified IMC and the modified SP methods have the advantage of a faster nominal set-point response without overshoot for unstable processes. The common characteristic of the above-mentioned, modified IMC and SP methods is the use of a nominal process model in their control structures, which is responsible for their good performance in this respect. Most existing, modified IMC and SP methods are restricted to the unstable processes in the form of a first-order rational part plus time delay, which is incapable of sufficiently well representing a variety of industrial and chemical unstable processes. Furthermore, the unmodeled dynamics that are usually present inevitably tend to deteriorate the control system performance. Recently, Rao and Chidambaram^{18,19} proposed a PID controller in series with a lead-lag compensator for the open-loop unstable, second-order plus time delay processes with/without a zero. The method is based on DS and set-point weighting is used to reduce the overshoot for servo response. Although their method has two tuning parameters, a significant improvement is gained in load disturbance rejection performance.

The PID controller design has been discussed extensively in the literature for both stable and unstable processes, but the design of a simple and robust controller with improved performance has not yet been fully achieved. The existing modern control hardware provides microprocessor implementation for a flexible combination of conventional control algorithms to achieve enhanced control performance. The PID controller in series with a lead-lag compensator (hereafter, PIDC controller) is a typical example. The main reason for using the PIDC controller is to provide improved performance without tribulation. Many authors^{2–4,10,18–19} have proposed this type of PIDC control strategy, as described in Eq. 1 below.

$$G_c = K_c \left(1 + \frac{1}{\tau_I s} + \tau_D s \right) \frac{1 + as}{1 + bs} \quad (1)$$

where K_c , τ_I , and τ_D are the proportional gain, integral time constant, and derivative time constant of the PID controller, respectively, and a and b are the compensator parameters.

Since all the IMC and DS approaches utilize some kind of model reduction techniques to convert the ideal feedback controller to the low order PID controller, an approximation error necessarily occurs. The performance of the resulting controller mainly depends on the controller order as well as the model reduction technique applied. A properly designed PIDC controller can mitigate the conversion error remarkably and provide a significant performance enhancement over the conventional PID controller.

The present study focused on the design of the second-order process in Eq. 2 to fulfill the various objectives: the tuning rule should be simple, analytically derived, model-based, and able to be applied to many process categories with a wide range of variation in process parameters in a unified framework.

$$G_p = \frac{(\tau_a s \pm 1) K e^{-\theta s}}{(\tau_1 s \pm 1)(\tau_2 s \pm 1)} \quad (2)$$

It should be noted that the proposed method is applicable for the second order stable/unstable process only with real poles as seen from Eq. 2. The proposed method is extended for the first-order delay integrating processes with dead time. The set-point filter is used to eliminate the overshoot in set-point response. The performance of the proposed PIDC controller has been compared with those of several prominent conventional PID and PIDC controllers in such a way that each controller is tuned to the same robustness level by evaluating the peak of the maximum sensitivity (M_s).

Controller Design Method

The closed-loop control block diagram is shown in Figure 1, where G_p is the process transfer function, \tilde{G}_p the process model, q the IMC controller, f_R the set-point filter, and G_c the transfer function of the controller.

For the nominal case (i.e., $G_p = \tilde{G}_p$), the set-point and disturbance responses in the IMC control structure can be simplified as:

$$y = G_p q f_R r + (1 - \tilde{G}_p q) G_p d \quad (3)$$

In accordance with the IMC method,² the process model \tilde{G}_p is factored into two parts:

$$\tilde{G}_p = p_m p_A \quad (4)$$

where p_m and p_A are the portions of the model inverted and not inverted by the controller, respectively, and $p_A(0) = 1$. The noninvertible part usually includes the dead time and/or right half plane zeros and is chosen to be all-pass.

Unstable Processes or Stable Processes with Poles Near Zero

To get a superior response for unstable processes or stable processes with poles near zero, the IMC controller q should satisfy following conditions.

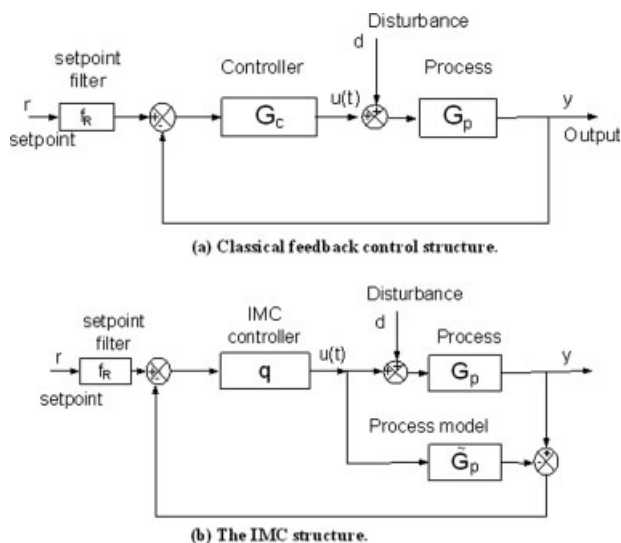


Figure 1. Block diagram of IMC and classical feedback control.

If the process G_p has unstable poles or poles near zero at z_1, z_2, \dots, z_m then

(i) q should have zeros at z_1, z_2, \dots, z_m

(ii) $1 - G_p q$ should also have zeros at z_1, z_2, \dots, z_m

Since the IMC controller q is designed as $q = p_m^{-1} f$ the first condition is satisfied automatically because p_m^{-1} is the inverse of the model portion with the unstable poles or poles near zero. The second condition can be fulfilled by designing the IMC filter f as:

$$f = \frac{\sum_{i=1}^m \alpha_i s^i + 1}{(\lambda s + 1)^r} \quad (5)$$

where λ is an adjustable parameter which controls the trade-off between the performance and robustness, r is selected to be large enough to make the IMC controller (semi-)proper, and α_i is determined by Eq. 6 to cancel the unstable poles or poles near zero in G_p .

$$1 - G_p q|_{s=z_1, \dots, z_m} = \left| 1 - \frac{p_A (\sum_{i=1}^m \alpha_i s^i + 1)}{(\lambda s + 1)^r} \right|_{s=z_1, \dots, z_m} = 0 \quad (6)$$

Then, the IMC controller is described as:

$$q = p_m^{-1} \frac{(\sum_{i=1}^m \alpha_i s^i + 1)}{(\lambda s + 1)^r} \quad (7)$$

Thus, the resulting set-point and disturbance responses are obtained as:

$$\frac{y}{r} = G_p q f_R = p_A \frac{(\sum_{i=1}^m \alpha_i s^i + 1)}{(\lambda s + 1)^r} f_R \quad (8)$$

$$\frac{y}{d} = (1 - G_p q) G_p = \left(1 - p_A \frac{(\sum_{i=1}^m \alpha_i s^i + 1)}{(\lambda s + 1)^r} \right) G_p \quad (9)$$

The numerator expression $(\sum_{i=1}^m \alpha_i s^i + 1)$ in Eq. 8 causes an excessive overshoot in the servo response, which can be eliminated by introducing the set-point filter f_R to compensate for the overshoot in the servo response.

From the above design procedure, a stable, closed-loop response can be achieved by using the IMC controller. The ideal feedback controller that is equivalent to the IMC controller can be expressed in terms of the internal model \tilde{G}_p and the IMC controller q :

$$G_c = \frac{q}{1 - \tilde{G}_p q} \quad (10)$$

Substituting Eqs. 4 and 7 into 10 gives the ideal feedback controller:

$$G_c = \frac{\frac{p_m^{-1} (\sum_{i=1}^m \alpha_i s^i + 1)}{(\lambda s + 1)^r}}{1 - \frac{p_A (\sum_{i=1}^m \alpha_i s^i + 1)}{(\lambda s + 1)^r}} \quad (11)$$

Controller Design for SOPDT Process

On the basis of the above design principle, the second-order plus dead time (SOPDT) process has been considered as a representative model:

$$G_p = \frac{K e^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)} \quad (12)$$

where K is the gain, τ_1 and τ_2 are the time constants, and θ the time delay. The IMC filter structure is chosen as

$$f = \frac{(\alpha_2 s^2 + \alpha_1 s + 1)}{(\lambda s + 1)^4} \quad (13)$$

The IMC filter form in Eq. 13 was also utilized by several researchers^{4,6,10} to design the IMC-PID controller. The resulting IMC controller becomes

$$q = \frac{(\tau_1 s + 1)(\tau_2 s + 1)(\alpha_2 s^2 + \alpha_1 s + 1)}{K(\lambda s + 1)^4} \quad (14)$$

Therefore, the ideal feedback controller is obtained as

$$G_c = \frac{(\tau_1 s + 1)(\tau_2 s + 1)(\alpha_2 s^2 + \alpha_1 s + 1)}{K[(\lambda s + 1)^4 - e^{-\theta s}(\alpha_2 s^2 + \alpha_1 s + 1)]} \quad (15)$$

Since the ideal feedback controller in Eq. 15 does not have the PID controller form, the remaining issue is to design the PID controller with a lead-lag compensator (i.e., PIDC controller) that approximates the ideal feedback controller most closely. The ideal feedback controller, G_c , equivalent to the IMC controller, can be obtained after the approximation of the dead time $e^{-\theta s}$ by 1/1 Pade expansion as

$$e^{-\theta s} = \frac{(1 - \theta s/2)}{(1 + \theta s/2)} \quad (16)$$

and results in

$$G_c = \frac{[(\tau_1 s + 1)(\tau_2 s + 1)](\alpha_2 s^2 + \alpha_1 s + 1)(1 + \theta s/2)}{K(\theta + 4\lambda - \alpha_1)s \left[1 + \frac{(\alpha_1 \theta/2 - \alpha_2 + 2\lambda\theta + 6\lambda^2)}{(\theta + 4\lambda - \alpha_1)}s + \frac{(\alpha_2 \theta^2/2 + 3\lambda^2\theta + 4\lambda^3)}{(\theta + 4\lambda - \alpha_1)}s^2 + \frac{2\lambda^3\theta + \lambda^4}{(\theta + 4\lambda - \alpha_1)}s^3 + \frac{\lambda^4\theta^2/2}{(\theta + 4\lambda - \alpha_1)}s^4 \right]} \quad (17)$$

As seen in Eq. 17, the resulting controller has the form of a PID controller cascaded with a high-order filter. The analytical PID formula can be obtained by rearranging Eq. 17 to give

$$K_C = \frac{\alpha_1}{K(4\lambda + \theta - \alpha_1)}; \quad \tau_I = \alpha_1; \quad \tau_D = \frac{\alpha_2}{\alpha_1} \quad (18)$$

Furthermore, it is obvious from Eq. 6 that the remaining part of the denominator in Eq. 17 contains the factor of the process poles $(\tau_1 s + 1)(\tau_2 s + 1)$. Therefore, the parameter b in Eq. 1 can be obtained by taking the first derivative of Eq. 19 below:

$$(cs^2 + bs + 1) = \frac{\left[1 + \frac{(\alpha_1 \theta/2 - \alpha_2 + 2\lambda\theta + 6\lambda^2)}{(\theta + 4\lambda - \alpha_1)}s + \frac{(\alpha_2 \theta^2/2 + 3\lambda^2\theta + 4\lambda^3)}{(\theta + 4\lambda - \alpha_1)}s^2 + \frac{(2\lambda^3\theta + \lambda^4)}{(\theta + 4\lambda - \alpha_1)}s^3 + \frac{\lambda^4\theta^2/2}{(\theta + 4\lambda - \alpha_1)}s^4 \right]}{(\tau_1 s + 1)(\tau_2 s + 1)} \quad (19)$$

and substituting $s = 0$ as

$$b = \frac{(\alpha_1 \theta/2 - \alpha_2 + 2\lambda\theta + 6\lambda^2)}{(\theta + 4\lambda - \alpha_1)} - (\tau_1 + \tau_2) \quad (20)$$

The parameter a in Eq. 1 can be easily obtained from Eq. 17 as

$$a = 0.5\theta \quad (21)$$

Since the high-order cs^2 term has little impact on the overall control performance in the control relevant frequency range, the remaining part of the fraction in Eq. 17 can be successfully approximated to a simple, first-order lead/lag compensator as $(1 + as)/(1 + bs)$.

The values of α_1 and α_2 are selected to cancel out the poles at $-1/\tau_1$ and $-1/\tau_2$. This requires $[1 - Gq]|_{s=-1/\tau_1, -1/\tau_2} = 0$ and thus $[1 - (\alpha_2 s^2 + \alpha_1 s + 1)e^{-\theta s}/(\lambda s + 1)^4]|_{s=-1/\tau_1, -1/\tau_2} = 0$. The values of α_1 and α_2 are obtained after simplification and given below.

$$\alpha_1 = \frac{\tau_1^2 \left[\left(1 - \frac{\lambda}{\tau_1} \right)^4 e^{-\theta/\tau_1} - 1 \right] - \tau_2^2 \left[\left(1 - \frac{\lambda}{\tau_2} \right)^4 e^{-\theta/\tau_2} - 1 \right]}{(\tau_2 - \tau_1)} \quad (22)$$

$$\alpha_2 = \tau_2^2 \left[\left(1 - \frac{\lambda}{\tau_2} \right)^4 e^{-\theta/\tau_2} - 1 \right] + \tau_2 \alpha_1 \quad (23)$$

A set-point filter is suggested to enhance the servo response, which is comprised of the lag term $(\alpha_2 s^2 + \alpha_1 s + 1)$, by eradicating the excessive overshoot and f_R is given as:

$$f_R = \frac{\gamma \alpha_1 s + 1}{(\alpha_2 s^2 + \alpha_1 s + 1)} \quad (24)$$

where $0 \leq \gamma \leq 1$. The extreme case with $\gamma = 0$ has no lead term in the set-point filter, which would cause a slow servo response. Note that γ can be adjusted online to obtain the desired speed of the set-point response.

In the simulation study, a widely accepted, weighting-type set-point filter is used for all methods, except for the proposed method, to eliminate the overshoot in the set-point response. This filter is given as:

$$f_R = (\gamma \tau_1 s + 1)/(\tau_1 \tau_D s^2 + \tau_1 s + 1)$$

where $0 \leq \gamma \leq 1$.

Regarding the classification of the process model to deal with the proposed design method, the unstable processes that do not have the form of Eq. 12 can be easily transformed to the form of Eq. 12 by adjusting their sign for controller design.

i. The controller design for the process model $G_p = Ke^{-\theta s}/(\tau_1 s \pm 1)(\tau_2 s \pm 1)$ can be extended in a similar fashion to that described above for the SOPDT design method.

ii. The first-order integrating process with time delay, $G_p = Ke^{-\theta s}/s(\tau s \pm 1)$, is approximated as a second-order process, $G_p = K\psi e^{-\theta s}/(\psi s \pm 1)(\tau s \pm 1)$, to design the proposed controller.

iii. The process with negative zero, $G_p = K(\tau_a s + 1)e^{-\theta s}/(\tau_1 s \pm 1)(\tau_2 s \pm 1)$, requires one extra filter, $1/(\tau_a s + 1)$. To keep the control structure in the form of Eq. 1, the filter, $(1 + as)/(1 + (b + \tau_a)s)$, is recommended instead of $(1 + as)/(1 + (b + \tau_a)s + b\tau_a s^2)$. The high-order term, $b\tau_a s^2$ has little impact on the overall control performance and can be neglected.

The positive zero in the second-order process is approximated as dead time to design the controller, i.e., $G_p = K(-\tau_a s + 1)e^{-\theta s}/(\tau_1 s \pm 1)(\tau_2 s \pm 1) = Ke^{-(\theta + \tau_a)s}/(\tau_1 s \pm 1)(\tau_2 s \pm 1)$. This approximation $(-\tau_a s + 1 \approx e^{-\tau_a s})$ is reasonable since an inverse response has a deteriorating effect on the control, similar to that of a time delay.

Remark 1. For the second-order, stable/unstable process without any zero, the designed value of b is too large for the closed-loop system to obtain robust performances when the parametric uncertainties are large. Based on extensive simulation study that has been conducted on different stable/unstable processes, a robust control performance was achieved by using a value of $0.1b$ instead of b .^{18,19}

Table 1. Controller Parameters and Resulting Performance Indices for Example 1

Method	K_c	τ_I	τ_D	Set-Point					Disturbance				
				Nominal Case			10% Mismatch		Nominal Case			10% Mismatch	
				ITAE	Overshoot	TV	ITAE	Overshoot	ITAE	Overshoot	TV	ITAE	Overshoot
Proposed method $\lambda = 1.182$	9.8092	5.4502	1.6898	10.89	1.009	13.58	11.28	1.004	3.50	0.089	1.70	3.49	0.091
Chen and Seborg $\lambda = 2.40$	6.3848	7.6045	2.0977	12.68	1.009	5.46	13.02	1.004	9.77	0.148	1.71	9.70	0.156
Shamsuzzoha and Lee $\lambda = 1.60$	6.4156	6.8593	1.9798	13.17	1.009	4.86	13.97	1.003	7.90	0.149	1.83	7.88	0.159

Proposed method, $\gamma = 0.3$, $f_R = (1.6351s+1)/(9.2099s^2+5.4502s+1)$, $a = 0.5$, $b = 0.0341$.
 Chen and Seborg, $\gamma = 0.5$, $f_R = (3.8022s+1)/(15.9520s^2+7.6045s+1)$.
 Shamsuzzoha and Lee, $\gamma = 0.4$, $f_R = (2.7437s+1)/(13.5800s^2+6.8593s+1)$.
 $M_s = 1.87$.

Simulation Study

This section deals with the simulation study conducted on the different types of representative model to cover several process classes. The performance of a classical PID controller can be significantly improved by adding a simple lead-lag compensator with a proper design method. To illustrate its superiority, the proposed PIDC controller has been compared with either the classical PID only controllers or the other existing PIDC controller with the same structure. The performance and robustness of the control system were evaluated using the following indices to ensure a fair comparison.

Integral error criteria

To evaluate the closed-loop performance, the integral of the time-weighted absolute error (ITAE) criterion was considered in the case of both a step set-point change and a step load disturbance. The ITAE is defined as:

$$ITAE = \int_0^{\infty} t|e(t)|dt \quad (25)$$

Overshoot

Overshoot is a measure of how much the response exceeds the ultimate value following a step change in the set-point and/or disturbance.

Maximum sensitivity (M_s) to modeling error

To evaluate the robustness of a control system, the maximum sensitivity, M_s , defined as $M_s = \max|1/[1 + G_p G_c(i\omega)]|$, is used. Since M_s is the inverse of the shortest distance from the Nyquist curve of the loop transfer function to the critical point $(-1, 0)$, a small M_s value indicates that the control system has a large stability margin. M_s is a well-known robustness measure that has been used by many researchers (Chen and Seborg,⁹ Skogestad,⁵ Shamsuzzoha and Lee⁶). To ensure a fair comparison, it is widely accepted for the model-based controllers (DS-d and IMC) to be tuned by adjusting λ so that the M_s values are the same. Therefore, in all our simulation examples (except Example 6), all of the controllers compared were designed to have the same robustness level in terms of the maximum sensitivity, M_s .

Total variation

To evaluate the manipulated input usage, we compute the total variation (TV) of the input $u(t)$ which is the sum of all its moves up and down. If we discretize the input signal as the sequence $[u_1, u_2, u_3, \dots, u_i, \dots]$, then $TV = \sum_{i=1}^{\infty} |u_{i+1} - u_i|$ should be minimized. TV is a good measure of the smoothness of a signal (Skogestad⁵).

Example 1. Stable SOPDT Process

Consider the following SOPDT process (Chen and Seborg,⁹ Shamsuzzoha and Lee⁶):

$$G_p = \frac{2e^{-1s}}{(10s+1)(5s+1)} \quad (26)$$

The proposed PIDC controller is compared with two other PID controllers: DS-d⁹ and the method of Shamsuzzoha and Lee⁶ (hereafter, SL method). The controller parameters, including the performance and robustness matrix, are listed in Table 1. In order to ensure a fair comparison, all of the compared controllers are tuned to have $M_s = 1.87$ by adjusting their respective λ .

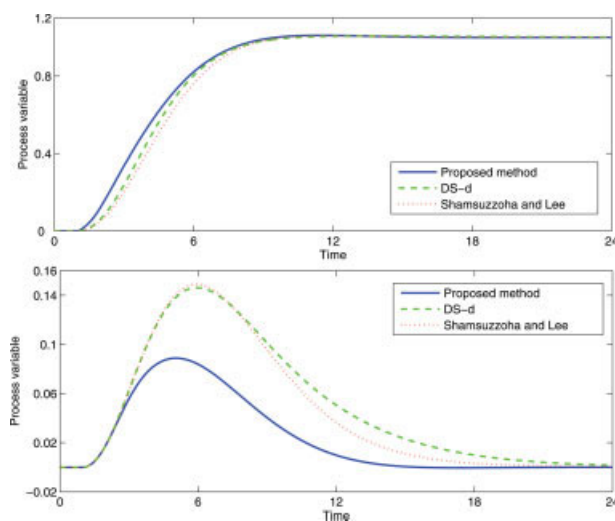


Figure 2. Response of the nominal system for Example 1.

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

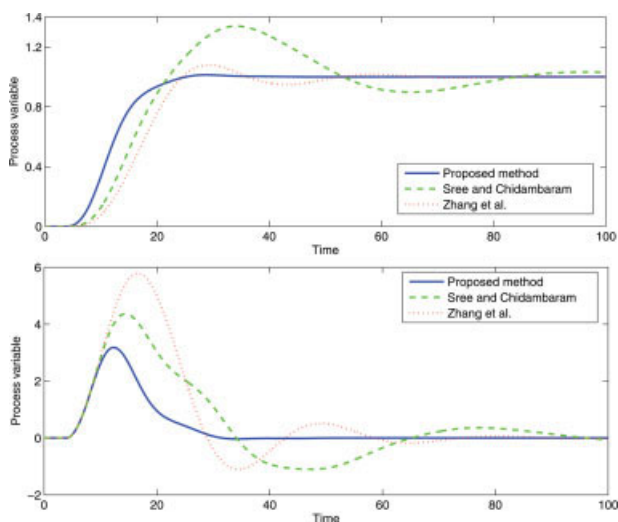


Figure 3. Response of the nominal system for Example 2.

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

A unit step change is introduced in both the set-point and load disturbance. Figure 2 compares the set-point and load disturbance responses obtained using three compared controllers. The 2DOF control scheme using the set-point filter was used in each method to enhance the set-point response. The proposed controller shows both smaller overshoot and faster disturbance rejection than those by the DS-d and SL methods. Although the overshoot is almost similar for the DS-d and SL methods, the DS-d method shows the slowest response and requires long settling time. The proposed controller shows significant advantages in overshoot and fast settling time, particularly in disturbance rejection. The closed-loop response for both the set-point tracking and disturbance rejection confirms the superior response of the proposed controller for the same robustness.

The robust performance is evaluated by simultaneously inserting a perturbation uncertainty of 10% in all three parameters in the worst direction and finding the actual process as $G_p = 2.2e^{-1.1s}/(9s + 1)(4.5s + 1)$. The simulation results for the model mismatch for the three methods are given in Table 1. The performance and robustness indices clearly demonstrate the superior robust performance of the proposed controller.

Example 2: First-Order Delay Integrating Process

Consider the first-order integrating process used by Zhang et al.²⁰ and Sree and Chidambaram.²¹

$$G_p = \frac{1e^{-4s}}{s(4s + 1)} \quad (27)$$

In the simulation study, we compare the proposed PIDC controller with the PIDC controller by Zhang et al.²⁰ and the PID controller by Sree and Chidambaram.²¹ For both the proposed method and that of Zhang et al.,²⁰ λ is adjusted to obtain $M_s = 3.28$ in order to ensure the same robustness with that of Sree and Chidambaram's²¹ method.

The proposed controller was designed by considering the above process as $G_p = 100e^{-4s}/(100s + 1)(4s + 1)$. Figure 3 shows the closed-loop output response for a unit step change in both the set-point and load disturbance. The controller setting parameters and performance matrix for both the set-point and load disturbance are listed in Table 2. To eliminate the overshoot in the set-point response, a set-point filter is used in each method, as provided in Table 2. The proposed controller has a small peak and fast settling time in the disturbance rejection, whereas in the set-point it has no overshoot and a faster settling time than those of the other controllers. This comparison of the output response and the values of the performance matrices listed in Table 2 confirms the superior performance of the proposed controller.

To confirm the robust performance of the proposed controller, 10% parameter perturbations were assumed in K , τ , and θ simultaneously towards the worst-case model mismatch and in finding the actual process as $G_p = 1.1e^{-4.4s}/s(3.6s + 1)$. The simulation results for the model mismatch for various methods are also given in Table 2. The robust performance indices demonstrate the superior robust performance of the proposed controller.

Example 3: SOPDT with Inverse Response

This example is frequently encountered in typical processes such as the boiler drum level control by the heating medium flow rate, the tray composition control of a distillation column by the vapor flow rate, and the exit temperature of a tubular exothermic reactor. The process exhibits inverse

Table 2. Controller Parameters and Resulting Performance Indices for Example 2

Method	K_c	τ_I	τ_D	Set-Point						Disturbance					
				Nominal Case			10% Mismatch			Nominal Case			10% Mismatch		
				ITAE	Overshoot	TV	ITAE	Overshoot	TV	ITAE	Overshoot	TV	ITAE	Overshoot	TV
Proposed method $\lambda = 2.117$	0.3593	12.130	2.704	87.31	1.012	0.393	128.9	1.050	499.4	3.179	3.36	953.1	3.741		
Sree and Chidambaram	0.199	10.0	7.02	560.0	1.339	0.292	478.6	1.335	3092.0	4.363	3.17	2835.0	4.960		
Zhang et al. $\lambda = 2.782$	0.2251	16.346	3.021	203.8	1.079	0.267	243.8	1.070	2109.0	5.778	3.49	3032.0	6.433		

Proposed method, $\gamma = 0$, $f_R = 1/(32.8106s^2 + 12.1304s + 1)$, $a = 2.0$, $b = 0.049$.

Sree and Chidambaram, $\gamma = 0$, $f_R = 1/(70.2s^2 + 10s + 1)$.

Zhang et al., $\gamma = 0$, $f_R = 1/(49.3845s^2 + 16.346s + 1)$, $b = 0.2966$.

$M_s = 3.28$.

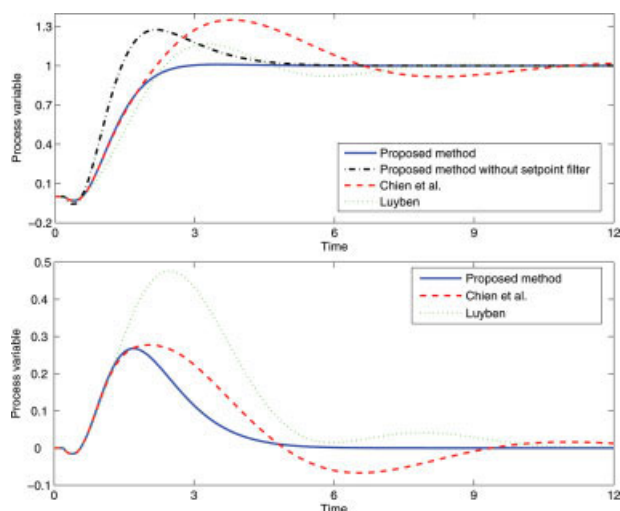


Figure 4. Response of the nominal system for Example 3.
[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

response when the initial response is in the opposite direction with respect to the ultimate steady-state value. It is an essential characteristic of the process with inverse response that the process transfer function has one or an odd number of zeros in the open right half plane.

An example that exhibits this characteristic has been studied by Luyben²² and Chien et al.²³ and is considered for the simulation study:

$$G_p = \frac{(-0.2s + 1)e^{-0.2s}}{(1s + 1)(1s + 1)} \quad (28)$$

The inverse response time constant (negative numerator time constant) can be approximated as a time delay such as $(-\theta_0^{\text{inv}}s + 1) \approx e^{-\theta_0^{\text{inv}}}$ for the proposed controller design. This is reasonable since an inverse response has a deteriorating effect on control, similar to that of a time delay (Skogestad⁵). Therefore, the above process can be approximated as the SOPDT model in Eq. 29 and the tuning parameters are estimated by the analytical rule previously proposed.

$$G_p = \frac{e^{-0.4s}}{(1s + 1)(1s + 1)} \quad (29)$$

Figure 4 shows the output response of the proposed PIDC controller and those of the PID controllers by Luyben²² and Chien et al.²³ for a unit step change in both the set-point and load disturbance. The PID controller settings are adjusted to obtain $M_s = 1.88$ for each method and the PID parameters with the performance matrix are listed in Table 3. The figure clearly shows the smaller overshoot and faster settling time of the proposed output response compared to that of the Chien et al.²³ method, while the Luyben²² method has an aggressive response with a significant overshoot that takes a long time to settle at a steady-state value. The performance values clearly indicate the advantage of the proposed controller over other controllers. For the servo response, the proposed method without a set-point filter shows a slightly large overshoot but a fast rising and settling time. The overshoot can be drastically minimized by using the set-point filter. Figure 4 and Table 3 clearly demonstrate the superiority of the proposed PIDC controller over the other PID controllers. The robustness of the controller is evaluated to be $G_p = (-0.23s + 1)1.15e^{-0.23s}/(0.85s + 1)(0.85s + 1)$, by considering the worst case under a 15% uncertainty in all four parameters. The simulation results for all design methods are given in Table 3. The error integral values for the proposed method prove to be the best.

Example 4. SODUP (One Stable Pole)

The following unstable process is considered for the present study (Lee et al.,¹⁰ Yang et al.,¹¹ Tan et al.¹³ and Rao and Chidambaram¹⁸):

$$G_p = \frac{e^{-0.939s}}{(5s - 1)(2.07s + 1)} \quad (30)$$

The above unstable process is transformed into Eq. 12 as $K = -1$, $\tau_1 = -5$, $\tau_2 = 2.07$, and $\theta = 0.939$ for the design of the proposed controller.

Rao and Chidambaram¹⁸ and Shamsuzzoha and Lee⁶ have already demonstrated the advantage of their methods over several other well-known design methods. Therefore, the proposed method is compared with these two to evaluate its relative performance. The λ value for the proposed and SL methods is adjusted to obtain the same value of $M_s = 2.34$ as for the method of Rao and Chidambaram.¹⁸ The controller setting parameters with performance indices are listed in Table 4.

Table 3. Controller Parameters and Resulting Performance Indices for Example 3

Method	K_c	τ_I	τ_D	Set-Point						Disturbance					
				Nominal Case			15% Mismatch			Nominal Case			15% Mismatch		
				ITAE	Overshoot	TV	ITAE	Overshoot	TV	ITAE	Overshoot	TV	ITAE	Overshoot	TV
Proposed method $\lambda = 0.443$	3.0819	1.6399	0.4295	1.882	1.274	6.0	2.120	1.499	1.17	0.267	2.053	1.207	0.367		
Chien et al. $\lambda = 0.091$	1.7182	1.0	1.0	7.805	1.350	3.65	5.689	1.351	3.75	0.277	2.177	3.034	0.343		
Luyben	1.390	1.95	—	3.272	1.167	1.99	4.179	1.278	4.71	0.476	1.575	4.541	0.580		

Proposed method $\gamma = 0.5$, $f_R = 1/(0.7044s^2 + 1.6399s + 1)$, $a = 2.0$, $b = 0.1715$.

A set-point filter is used only for Figure 4; performance indices in Table 3 are based on without a set-point filter.

Chien et al., no set-point filter is used.

Luyben, no set-point filter is used.

$M_s = 1.88$.

Table 4. Controller Parameters and Resulting Performance Indices for Example 4

Method	K_c	τ_I	τ_D	Set-Point					Disturbance				
				Nominal Case			10% Mismatch		Nominal Case			10% Mismatch	
				ITAE	Overshoot	TV	ITAE	Overshoot	ITAE	Overshoot	TV	ITAE	Overshoot
Proposed method $\lambda = 0.9296$	6.7051	5.4738	1.3330	7.929	1.030	15.96	12.40	1.052	4.365	0.163	2.54	5.94	0.192
Rao and Chidambaram $\lambda = 1.50$	6.4285	6.4409	1.4135	6.762	1.024	94.39	12.70	1.055	5.921	0.166	3.41	7.01	0.190
Shamsuzzoha and Lee $\lambda = 1.71$	4.009	8.0327	1.6808	24.44	1.032	2.81	24.69	1.008	14.40	0.337	2.83	13.52	0.387

Proposed method, $\gamma = 0.3$, $f_R = (1.6421s+1)/(7.2966s^2+5.4738s+1)$, $a = 0.4695$, $b = 0.023$.
Rao and Chidambaram, a set-point weighting parameter is set as 0.39, $a = 0.4695$, $b = 0.0152$.
Shamsuzzoha and Lee, $\gamma = 0$, $f_R = 1/(13.5014s^2+8.0327s+1)$.
 $Ms = 2.34$.

For the simulation of the above process, both the load disturbance, with a step change of magnitude 1, and the set-point are inserted and the output response is shown in Figure 5. The proposed controller has a faster settling time than the other controllers do.

Table 4 also presents the performance index, with 10% uncertainties in gain and dead time, and a time constant of $G_p = 1.1e^{-1.0329s}/(4s-1)(1.863s+1)$ toward the worst-case model mismatch. The proposed controller clearly shows a better response for both set-point tracking and disturbance rejection for the model mismatch case.

Example 5. SODUP (Two Unstable Poles)

The following widely published SODUP example has been considered for the comparison (Rao and Chidambaram,¹⁸ Liu et al.¹⁴):

$$G_p = \frac{2e^{-0.3s}}{(3s-1)(1s-1)} \quad (31)$$

The recently published papers of Rao and Chidambaram¹⁸ and Liu et al.¹⁴ demonstrated the superiority of their methods

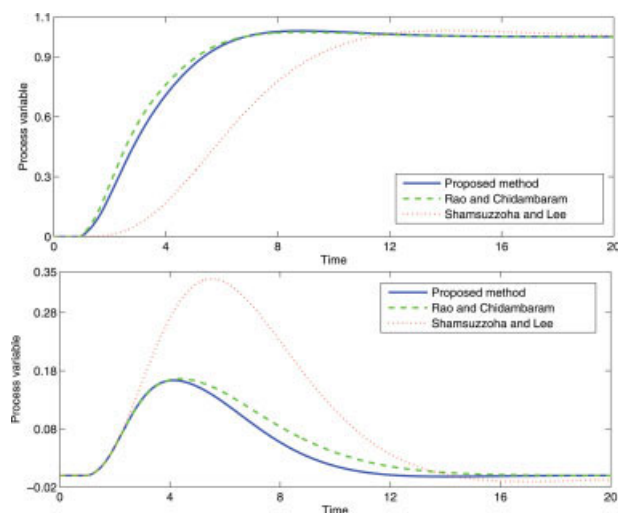


Figure 5. Response of the nominal system for Example 4.

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

for the SODUP over several widely accepted, previous approaches. Therefore, the proposed method is compared with those of Rao and Chidambaram¹⁸ and Liu et al.¹⁴ For a fair comparison, λ for the proposed method is adjusted to give the same $Ms = 3.09$ as for Rao and Chidambaram¹⁸ and the tuning parameters are listed in Table 5. For the Liu et al.¹⁴ method, a high-order controller is used for the comparison because they demonstrated a significant performance improvement by using the high-order controller. Figure 6 compares the proposed PIDC controller with the PIDC controller by Rao and Chidambaram¹⁸ and the high order controller by Liu et al.,¹⁴ by introducing a unit step change in the set-point and the load disturbance, respectively. For the servo response, the set-point filter is used for both the proposed and Rao and Chidambaram¹⁸ methods, whereas the three-control-element structure is used for the Liu et al.¹⁴ method. It is important to note that the well-known, modified IMC structure has the theoretical advantage of eliminating the time delay from the characteristic equation. Unfortunately, this advantage is lost if the process model is inaccurate. Besides, real process plants usually incorporate unmodeled dynamics that inevitably tend to deteriorate the control system performance severely.

Figure 6 and performance indices listed in Table 5 clearly show significantly improved load disturbance response and fast settling time of the proposed method. All three methods exhibit similar servo responses.

The robustness of the controller was investigated by inserting a perturbation uncertainty of 5% in all three parameters simultaneously towards the worst-case model mismatch, i.e., $G_p = 2.1e^{-0.315s}/(2.85s-1)(0.95s-1)$. The simulation results presented in Table 5 for both the set-point and the disturbance rejection clearly show the excellent set-point and load response of the proposed PIDC controller. The high order controller by Liu et al.¹⁴ has the worst performance for both set-point and disturbance rejection response for the model mismatch.

Remark 2. It is worth while to investigate which part of the proposed method mainly contributes to the enhanced performance in comparison with the Rao and Chidambaram¹⁸ method that has the same PIDC structure.

Most important factor for the performance improvement is the selection of the desired closed loop transfer function. In the proposed method, the 4th order IMC filter given in Eq. 13 is selected while the desired closed loop transfer function

Table 5. Controller Parameters and Resulting Performance Indices for Example 5

Method	K_c	τ_i	τ_D	Set-Point					Disturbance				
				Nominal Case			5% Mismatch		Nominal Case			5% Mismatch	
				ITAE	Overshoot	TV	ITAE	Overshoot	ITAE	Overshoot	TV	ITAE	Overshoot
Proposed method $\lambda = 0.3555$	3.4706	1.5052	1.3633	1.103	1.023	11.66	1.254	1.025	0.86	0.239	4.46	0.901	0.229
Rao and Chidambaram $\lambda = 0.64$	2.9723	1.9003	1.6141	1.032	1.020	93.28	1.378	1.029	1.51	0.259	7.04	1.583	0.234
Liu et al. $\lambda = 0.51$	—	—	—	1.267	1.001	9.76	1.417	1.022	1.67	0.379	6.41	1.886	0.372

Proposed method, $\gamma = 0.3$, $f_R = (0.4516s+1)/(2.0519s^2+1.5052s+1)$, $a = 0.15$, $b = 0.0059$.

Rao and Chidambaram, a set-point weighting parameter is set as 0.536, $a = 0.15$, $b = 0.0041$.

Liu et al., two-degree-of-freedom control structure with three controllers:

A disturbance estimator $G_c = \frac{32.82s^3+439.41s^2+232.64s+129.79}{0.56s^3+0.8s^2+100s}$, a set-point controller $k_d = 3.0$, $C(s) = \frac{1.5s^2+s+0.5}{(0.51s+1)^2}$.
 $Ms = 3.09$.

used in the Rao and Chidambaram¹⁸ method is equivalent to that by the 3rd order IMC filter as $f = (\alpha_2 s^2 + \alpha_1 s + 1)/(\lambda s + 1)^3$ in the IMC approach. Since both the IMC and DS approaches utilize some kind of model reduction techniques to convert the ideal feedback controller to the PID controller, a conversion error necessarily occurs. Therefore, for a given process model, there exists an optimum IMC filter structure which results in the best PID performance.⁶ For a given IMC filter, down to some optimum λ value, the resulting PID controller have no significant conversion error and thus a little difference with the ideal controller in performance, and after some minimum IAE point the conversion error rises sharply towards unstable limits. Our investigation shows that a high order filter structure tends to allow the resulting PID controller to keep a reasonably small conversion error up to a smaller λ value than a low order filter structure and thus gives an enhanced PID performance. This is why much smaller λ value could be used in the proposed method while a larger λ should be used in the Rao and Chidambaram.

It is another contribution factor the way to determine the PID parameters from the ideal controller. As seen from Eqs. 22 and 23, no approximation is introduced to obtain α_1 and α_2 for more accurate pole cancellation while the Pade approximation is assumed in the Rao and Chidambaram method to determine the PID parameters. In the case where the IMC filter $f = (\alpha_2 s^2 + \alpha_1 s + 1)/(\lambda s + 1)^3$ with $\lambda = 0.64$ was applied to the proposed approach so that the resulting desired closed loop transfer function became same as that by the Rao and Chidambaram¹⁸ method, the resulting PIDC controller still gave an improved performance.

Example 6. SODUP with Negative Zero

Consider the following unstable process with a strong lead-time constant and two unstable poles:

$$G_p = \frac{2(5s+1)e^{-0.3s}}{(3s-1)(1s-1)} \quad (32)$$

The proposed controller is compared with those of Rao and Chidambaram¹⁹ and Lee et al.¹⁰ The controller setting parameters of Rao and Chidambaram¹⁹ are obtained from their published paper. For the method of Lee et al.¹⁰, the PID setting with $\lambda = 0.60$ is used for a smooth and fast response because the original PID setting with $\lambda = 0.45$ reported in their paper yields too oscillatory output response. A value of $\lambda = 0.3$ is

selected for the proposed method. The controller setting parameters are listed in Table 6. Figure 7, showing the closed-loop output response for a unit-step set-point change for both the servo and load disturbance, demonstrates the excellent enhancement of the proposed controller for both the servo and regulatory problem. For the servo response, each one utilizes a set-point filter to reduce the overshoot in the set-point response. In both the set-point and disturbance response the proposed controller shows a fast settling time.

The robust performance is evaluated by simultaneously inserting a perturbation uncertainty of 5% in all three parameters in the worst direction and finding the actual process with $G_p = 2.1(5.25s+1)e^{-0.315s}/(2.85s-1)(0.95s-1)$. The simulation results for the model mismatch case are also provided in Table 6. The proposed controller has a superior robust performance compared to that by Lee et al.¹⁰ whereas the response by Rao and Chidambaram¹⁹ becomes unstable.

Example 7. Different Disturbance Dynamics

The proposed method is based on the case where the disturbance is introduced into the process through G_p as depicted in Figure 1. However, it can occur that the distur-

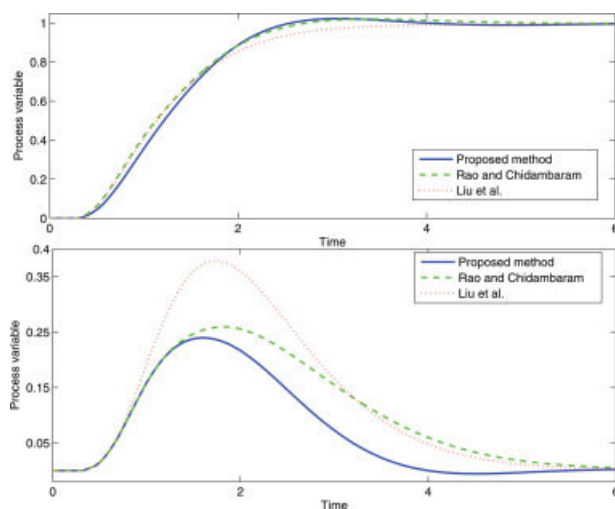


Figure 6. Response of the nominal system for Example 5.

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

Table 6. Controller Parameters and Resulting Performance Indices for Example 6

Method	K_c	τ_I	τ_D	Set-Point					Disturbance				
				Nominal Case			5% Mismatch		Nominal Case			5% Mismatch	
				ITAE	Overshoot	TV	ITAE	Overshoot	ITAE	Overshoot	TV	ITAE	Overshoot
Proposed method $\lambda = 0.3$	4.6264	1.3537	1.1093	1.369	1.007	1.137	1.852	1.043	3.023	1.678	5.06	9.367	2.014
Rao and Chidambaram $\lambda = 0.3$	3.1157	3.7054	2.6248	14.11	0.708	0.69	Unstable	–	7.364	1.524	11.35	Unstable	–
Lee et al. $\lambda = 0.60$	1.1902	1.9297	3.1224	4.656	1.006	0.773	6.14	1.043	13.63	3.337	7.087	38.55	3.804

Proposed method, $\gamma = 0$, $f_R = 1/(1.5016s^2 + 1.3537s + 1)$, $a = 0.15$, $b = 5.0453$.

Rao and Chidambaram, a set-point weighting parameter is set as 0.0918, $a = 0.15$, $b = 6.3594$.

Lee et al., $\gamma = 0$, $f_R = 1/(5.9571s^2 + 1.8894s + 1)$, $b = 4.9890$.

ance dynamics G_D might be different from the process dynamics G_p as shown in Figure 8. It is worthwhile to investigate the validity of the proposed method when the disturbance dynamics is uncertain or different from G_p .

Consider a process described by the same transfer function as in Example 1 but with different disturbance transfer functions as follows:

$$G_D = \frac{4e^{-2s}}{(20s + 1)(10s + 1)} \quad (33)$$

$$G_D = \frac{1e^{-0.5s}}{(5s + 1)(2.5s + 1)} \quad (34)$$

The above two different transfer functions were obtained by increasing and decreasing process parameters by 100% from their original values in Eq. 26, respectively. Each of the two cases depicts a significant difference in G_p and G_D .

The PID controllers were designed based on G_p assuming that $G_p = G_D$ and thus taken from Table 1. Because G_D does not affect the set-point response, only the disturbance responses were simulated. Figure 9a,b compares the disturb-

ance responses by three compared controllers for the cases of Eqs. 33 and 34, respectively. The results shown in the figure confirms that the proposed PIDC controller has a smaller peak value and faster settling time in both cases than the other PID only controllers.

Remark 3. Closed-loop time constant (λ) guidelines

The closed-loop time constant λ is the only user-defined tuning parameter in the proposed tuning rule. It is directly related to the performance and robustness of the proposed controller and it is important to set some λ guidelines in order to provide both a fast and robust performance. The small peak and fast response of disturbance rejection are favored by selecting a small value of λ ; however, robustness and stability of the system require a sufficiently large value of λ . The extensive simulation studies presented here suggest that robust control performance can be achieved by considering the starting value of λ to be equal to the process time delay. If the selected λ does not satisfy the performance and robust stability criteria, then λ is monotonously increased or decreased on-line until the desirable trade-off between the close-loop nominal performance and its robust stability is achieved.

Conclusions

In this article, we have discussed a IMC based simple design method for a PIDC controller, i.e., a PID cascaded with a lead-lag compensator. The proposed method covered a broad class of second-order stable and unstable processes with time delay. Several important representative processes were considered in the simulation study in order to demonstrate the superiority of the proposed method. The design method was based on the disturbance rejection and a set-point filter was suggested to eliminate the overshoot in set-

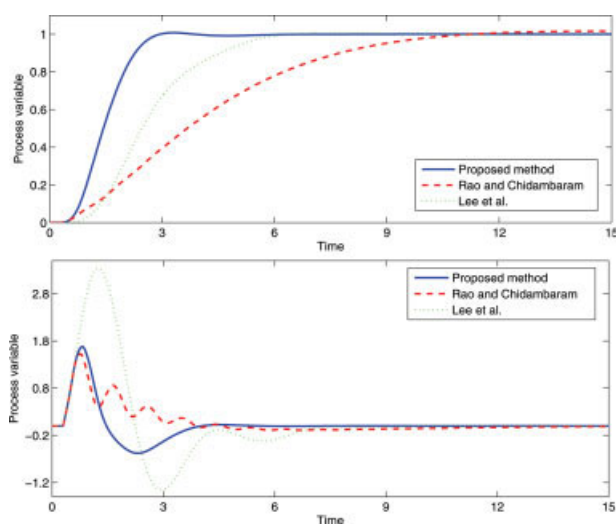


Figure 7. Response of the nominal system for Example 6.

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

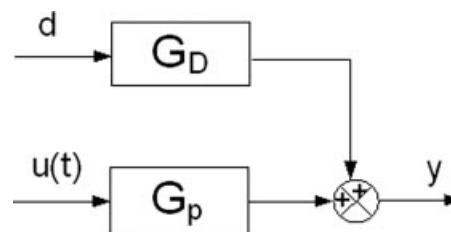


Figure 8. Disturbance into the process output ($G_p \neq G_D$).

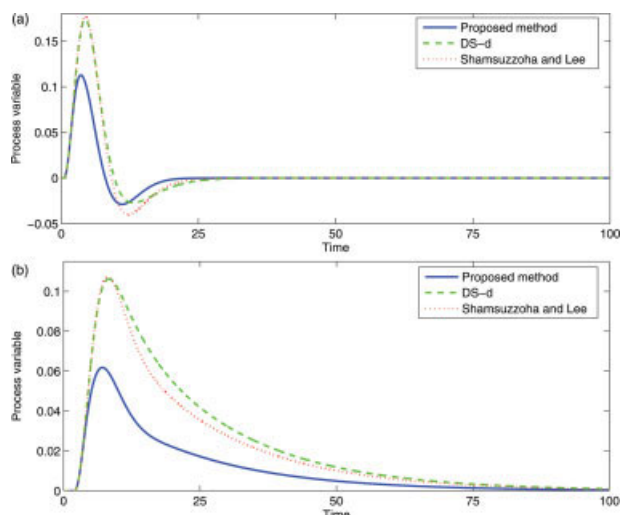


Figure 9. Response for the case of $G_p \neq G_D$ for Example 6.

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

point response. The results showed that both nominal and robust performances of the PID controller can be significantly enhanced by adding a simple lead-lag compensator with a proper design method. The proposed controller consistently achieved superior performance for several process classes. In the robustness study conducted by simultaneously inserting a perturbation uncertainty in all parameters in order to obtain the worst-case model mismatch, the proposed PIDC controller was found to be superior to the other PID and PIDC controllers.

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